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AN EXAMINATION OF METHODS OF BUFFETING ANALYSIS BASED ON EXPERIMENTS WITH WINGS OF VARYING STIFFNESS

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AN EXAMINATION OF METHODS OF BUFFETING ANALYSIS BASED ON EXPERIMENTS WITH WINGS OF VARYING STIFFNESS

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SUMMARY

An examination of the validity of some of the assumptions used in the analysis of buffeting loads has been made by means of wind-tunnel tests of models differing in stiffness. A linear analysis predicts buffeting loads which are high by about 25 percent. This difference may be associated with a relieving effect which causes flexible wings to generate smaller buffeting forces than a more rigid wing. A prediction based on aerodynamic damping only, which apparently contained compensating inaccuracies, provided values of buffeting loads which were closer to the measured values than those predicted by the more complete analysis including both structural and aerodynamic damping.

INTRODUCTION

Recent applications of the methods of generalized harmonic analysis to the problems of airplane buffeting, such as those contained in references 1 to 6, have demonstrated the usefulness of these methods in making a rational approach to the solution of these problems. For example, Liepmann in reference 3 has indicated from dimensional considerations that for certain conditions the buffeting loads experienced by an airplane should vary by the square root of the dynamic pressure instead of the direct variation found for steady loads. This result arises because of the importance of aerodynamic damping in limiting the response of the airplane to the fluctuating forces which cause buffeting. Similarly, dimensional analysis indicates that, if wings of different stiffness were subjected to the same aerodynamic buffeting conditions, the deflections of the wing would vary inversely as the square root of the stiffness.

The foregoing results are based on a somewhat simplified and idealized representation of the complex phenomenon of buffeting. The aerodynamic damping used is considered to be linear, that is, the coefficient of damping is a constant for all amplitudes of oscillation. When it is recalled that buffeting occurs at conditions where steady force coefficients are nonlinear, it seems unlikely that the assumption of linear characteristics during buffeting would be valid. In addition to the assumption of a constant coefficient of aerodynamic damping for various amplitudes of oscillation, the analyses presented in references 1 to 6 assume that the fluctuating forces which cause buffeting can be treated as external forces which are not affected by the resulting oscillations. Again, it seems quite possible that the separated flow which produces these driving forces could be affected by the motion of the wing. In the present investigation, an attempt has been made to shed further light on the question of the validity of these two assumptions.

The experiment was designed so that the many parameters which are of importance in buffeting, such as Mach number, Reynolds number, and reduced frequency, were held essentially constant while, for the same conditions, different amplitudes of response could be obtained. These conditions were accomplished by employing three cantilever-mounted semispan-wing models having identical geometry but constructed of steel, aluminum, and magnesium alloys. Use of the three different materials provided a range of stiffness of about 4.6 to 1. That is, for the same static load the magnesium wing would deflect about four times as much as the steel wing or, if the assumptions used in buffeting analyses are valid, the magnesium wing would deflect during buffeting about twice as much as the steel wing. An example of such an analysis is given in an appendix by Don D. Davis, Jr., of the Langley Research Center, which presents the derivation of the equations governing the buffet response of a wing.

The purpose of this paper is to present the results of measurements of the buffeting loads on these three wings and to interpret the results in terms of the assumptions involved in the application of generalized harmonic analysis to the study of buffeting loads.

SYMBOLS

$|A(\omega)|^2$ square of absolute value of system admittance

$[A]$ matrix of flexibility influence coefficients

$C_{L_{\alpha},1}$ first-mode generalized lift-curve slope for damping component of aerodynamic force due to the wing vibration,

$$\frac{\sum_{m} k_m s_m \left(\varphi_m^{(1)}\right)^2}{S_2}, \text{ per radian}$$

$C_{N,1}$ generalized normal-force coefficient for first-mode vibration, N_1/qS_1

$c(y)$ chord at any station y, ft

$\bar{c}$ average chord, ft

E elastic strain energy, lb-ft

F_e effective value of aerodynamic damping coefficient

f frequency of bending mode

g_1 structural damping coefficient in first bending mode

k reduced frequency, $\omega c/2V$

k_m constant relating the damping component of local pressure differential due to wing vibration to local angle of attack (in radians) and free-stream dynamic pressure

L_1 generalized damping constant for first-mode wing vibration, lb-sec/ft

l semispan of wing, ft

$\lceil M \rfloor$ diagonal inertia matrix for wing

$M_{m,1}$ effective moment (for first-mode vibration) of mass outboard of point y_g, $\int_{y_g}^{l}\left(y - y_g\right)m(y)w_1(y)\,dy$, slug-ft

M_n generalized wing mass for nth-mode vibration,

$$\sum_m m_m\left(\varphi_m^{(1)}\right)^2 \text{ or } \int_{-l}^{l} m(y)w_n^2(y)\,dy, \text{ slugs}$$

m_m mass of an element of wing, slugs

$m(y)$ spanwise mass distribution, slugs/ft

N_1 time-dependent generalized (for first-mode vibration) buffet force acting on wing, $\sum \Delta p_m s_m \varphi_m^{(1)}$, lb

$\{P\}$ column matrix representing a set of static loads applied to wing

P_m forces, other than inertia and elastic, that act on an element m

Δp_m local pressure difference (between bottom and top surfaces of wing) that excites the buffet vibration

q free-stream dynamic pressure, lb/sq ft

r_n time-dependent displacement of wing element for which $\varphi^{(n)} = 1$

S_1 weighted wing area for first-mode bending, $\sum s_m \varphi_m^{(1)}$ or $2\int_0^l c(y)w_1(y)\,dy$, sq ft

S_2 weighted wing area for first-mode vibration, $\sum_m s_m\left(\varphi_m^{(1)}\right)^2$ or $2\int_0^l c(y)w_1^2(y)\,dy$, sq ft

s_m area of mth element of wing, sq ft

T kinetic energy, lb-ft

t time, sec

$[U]$ dynamic matrix for wing

V free-stream velocity, ft/sec

$w_n(y)$ deflection of wing elastic axis normalized to unit deflection at wing tip

y spanwise coordinate

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y_g spanwise strain-gage location, ft

y_m a spanwise center of mass, $\dfrac{\int_{y_g}^{l} \left(y - y_g\right) m(y) w_1(y)\, dy}{\int_{-l}^{l} m(y) w_1^2(y)\, dy}$, ft

$\{z\}$ column matrix representing a set of vertical displacements of wing

z deflection of an element of wing

z_m time-dependent displacement of mth wing element, ft

a, b, c constants

α angle of attack, deg or radians

β phase angle by which displacement lags the force

γ_1 aerodynamic damping coefficient in first bending mode, $\dfrac{C_{L_{\alpha,1}} q S_2}{2 M_1 \omega_1 V}$

κ mass ratio parameter

ρ material density, slugs/cu ft

σ root-mean-square bending moment, in-lb

$\Phi_N(\omega)$ power spectral density of generalized normal force, $(\text{lb})^2$/cycle/sec

$\Phi_{r_1}(\omega)$ power spectral density of time-dependent displacement of wing element for which $\varphi^{(n)} = 1$

$\left\{\varphi^{(1)}\right\}$ column matrix representing a set of normalized deflections

$\varphi_m^{(n)}$ normalized deflection of mth wing element for wing vibration in nth normal mode

ω circular frequency, $2\pi f$, radians/sec

Subscripts:

A refers to aluminum wing

M refers to magnesium wing

S refers to steel wing

n refers to natural mode, where n is any integer

m refers to element of wing

1 refers to first mode

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Dots over a symbol denote differentiation with respect to time.

APPARATUS AND TECHNIQUE

Wind Tunnel

The tests were made in the Langley 2- by 4-foot flutter research tunnel. This tunnel is a conventional closed-throat, single-return wind tunnel capable of operation in either air or Freon-12 at stagnation pressures from 1 atmosphere down to approximately 1/10 atmosphere. The experiments reported herein were conducted in air at approximately atmospheric stagnation pressure.

Model Characteristics

The models used in this investigation were semispan, cantilever-mounted wings. Each had NACA 65A004 airfoil sections, was unswept about the midchord, had an aspect ratio of 3.8, and had a taper ratio of 0.225. A sketch of the models giving pertinent dimensions is shown in figure 1. The models were constructed of steel, aluminum alloy, and magnesium alloy. The first natural bending frequencies of the steel, magnesium, and aluminum models were 117.9 cycles per second, 119.9 cycles per second, and 126.8 cycles per second, respectively.

The structural damping coefficients for each of the wing models were obtained from the rate of decay of free vibrations in still air. Figure 2 shows the variation of structural damping in still air with root-mean-square bending moment. The three sets of values for the steel model were obtained from separate installations of the model in its mount and indicate a possible effect of root clamping on the effective structural damping. Structural damping coefficients were also obtained for the steel model subjected to a preload simulating the static aerodynamic loading present during the buffeting tests. It was found that there was no appreciable change in structural damping due to the preload.

Instrumentation

A schematic diagram of the instrumentation used is shown in figure 3. A resistance-wire strain-gage bridge was attached to the surface near the root of each of the cantilever-mounted models. The strain-gage bridge was arranged to be sensitive to bending strains only. A balance box was used for supplying direct current to the bridge. The output of the bridge was amplified by two linear amplifiers connected in series. The amplified signal was then passed through a vacuum-bulb thermocouple which converted the voltages proportional to the fluctuating strains into direct current proportional to the mean-square value of the fluctuating strains. This direct-current output of the thermocouple was read on a heavily damped microammeter. This system had a flat response within about 5 percent between frequencies of about 5 cycles per second and 5,000 cycles per second. The low-frequency cutoff of the linear amplifiers effectively subtracted the static value of the strain from the signal; that is, the strain due to the static angle of attack was removed and the mean-square values measured refer to fluctuations in strain falling in the frequency band between about 5 cycles per second and 5,000 cycles per second.

The reading of the microammeter was converted to a root-mean-square value of the bending moment by a system of calibrations. The relationship between the bending moment applied to the models and the voltage out of the strain-gage bridge was determined by applying a known static bending moment and reading the corresponding bridge output or unbalance on a self-balancing potentiometer. The relationship between voltage applied to the input of the linear amplifiers and current out of the thermocouple was determined by applying a known value of alternating voltage to the input of the linear amplifiers and reading the corresponding direct-current output of the thermocouple on the microammeter.

In order to check the overall accuracy of the system, an electromagnetic shaker was used to drive the models at their first natural frequency. The root-mean-square bending moment and tip amplitude were measured. The root-mean-square bending moments calculated from the measured tip amplitudes are compared with the measured root-mean-square bending moments for the aluminum-alloy model in figure 4. The agreement between the measured and calculated values is considered to be good.

Procedure

The linear amplifier and thermocouple were calibrated before and after each test to minimize the effects of a small change in sensitivity. The tunnel-off instrument noise level was also read before and after each run so that it could be extracted from the data as a tare.

With the model set at the desired angle of attack, the tunnel velocity was increased until the microammeter indicated at least twice the tunnel-off instrument noise reading. The microammeter reading and tunnel conditions were then recorded. These observations were repeated at several velocities. The tests covered a range of velocities from 160 to 640 feet per second and a range of angles of attack from 0° to 20°. Shown in figure 5 are the variations with velocity of Mach number, Reynolds number based on average chord, density, and reduced frequency based on average chord and the first bending frequency.

METHOD OF ANALYSIS

Since the purpose of this investigation is to examine the validity of some of the assumptions used in the analysis of buffeting loads, it would seem appropriate to state these assumptions and to present the pertinent results of these analyses. The development of these relationships has been presented in references 1 to 6 and the particular relationship of interest in this investigation is developed in the appendix.

The treatment of buffeting as the response of a linear single-degree-of-freedom system to a stationary random excitation has been discussed in references 1 to 6. These assumptions lead to the following expression (eq. (A10)) for the mean-square bending moment acting at a station near the root of the wing:

$$\sigma^2 = y_{m_1}^2 \Phi_N(\omega_1) \frac{\pi\omega_1}{4\left(\gamma_1 + \frac{1}{2}g_1\right)} \tag{1}$$

where γ_1 is the coefficient of aerodynamic damping in the first bending mode and g_1 is the coefficient of structural damping in the first bending mode. The term y_{m_1} has the dimensions of length and can be thought of as the distance from the strain-gage station to a spanwise center of mass which will be the same for the three models considered in the present investigation. The term $\Phi_N(\omega_1)$ represents the value of the power spectral density of the exciting force at the fundamental bending frequency and has the units of $(\text{force})^2$-second. The excitation will also be assumed to be the same for the three models at a given angle of attack and velocity. This is one of the assumptions that can be examined by the methods of the present investigation.

Thus, for models that differ only in the material of construction

$$\sigma^2 \propto \frac{\omega_1}{\gamma_1 + \frac{1}{2}g_1} \tag{2}$$

or

$$\sigma \propto \sqrt{\frac{\omega_1}{\gamma_1 + \frac{1}{2}g_1}} \tag{3}$$

Generally, the analysis of buffeting has been used to determine the relationship between the buffeting loads measured on one model and those to be expected on another model or on the full-scale airplane. For the conditions of the present investigation equation (3) can be used to give the following:

$$\left.\begin{aligned} \frac{\sigma_M}{\sigma_S} &= \sqrt{\frac{\omega_{1,M}}{\omega_{1,S}} \frac{\left(\gamma_1 + \frac{1}{2}g_1\right)_S}{\left(\gamma_1 + \frac{1}{2}g_1\right)_M}} \\ \frac{\sigma_A}{\sigma_S} &= \sqrt{\frac{\omega_{1,A}}{\omega_{1,S}} \frac{\left(\gamma_1 + \frac{1}{2}g_1\right)_S}{\left(\gamma_1 + \frac{1}{2}g_1\right)_A}} \end{aligned}\right\} \tag{4}$$

where the right-hand side of equations (4) may be thought of as representing the calculated scale factor required to convert the measured loads on the steel model (subscript S) to the predicted loads on the magnesium (subscript M) or aluminum model (subscript A).

In order to obtain values of the calculated scale factor, it will be necessary to evaluate the aerodynamic damping coefficient γ_1 which is defined as

$$\gamma_1 = \frac{\kappa F_e}{k_1} \tag{5}$$

where κ is a mass ratio parameter, k_1 is the reduced frequency based on the first natural frequency, and F_e is, as defined in reference 7, an effective section derivative having the same form as Theodorsen's circulation function $F(k)$ (see ref. 8). Values of F_e have been measured and are presented in reference 7 as functions of angle of attack and reduced frequency for two wings oscillating in the first bending mode. Values of the calculated scale factor have been determined for the models used in the present investigation by using measured values of the natural frequency and structural damping and values of F_e from reference 7.

Another form of the calculated scale factor is obtained from equation (4) when the structural damping is assumed to be small relative to the aerodynamic damping. For this case the scale factor is

$$\left.\begin{aligned} \frac{\sigma_M}{\sigma_S} &= \frac{\omega_{1,M}}{\omega_{1,S}}\sqrt{\frac{\rho_M}{\rho_S}} \\ \frac{\sigma_A}{\sigma_S} &= \frac{\omega_{1,A}}{\omega_{1,S}}\sqrt{\frac{\rho_A}{\rho_S}} \end{aligned}\right\} \tag{6}$$

where ρ is the density of the material of construction. This expression results in a single number for each of the models, regardless of test conditions. These values are

$$\frac{\sigma_M}{\sigma_S} = 0.490$$

$$\frac{\sigma_A}{\sigma_S} = 0.653$$

If it is assumed that the aerodynamic damping is small compared with the structural damping, the calculated scale factor for this case is simply

$$\left.\begin{aligned} \frac{\sigma_M}{\sigma_S} &= \sqrt{\frac{\omega_{1,M}}{\omega_{1,S}}\frac{g_{1,S}}{g_{1,M}}} \\ \frac{\sigma_A}{\sigma_S} &= \sqrt{\frac{\omega_{1,A}}{\omega_{1,S}}\frac{g_{1,S}}{g_{1,A}}} \end{aligned}\right\} \tag{7}$$

Since the structural damping coefficients for the models have been observed to vary with the amplitude of the oscillation (see fig. 2), the scale factor will also vary with the magnitude of the buffeting loads.

These three relationships (aerodynamic and structural damping, eq. (4); aerodynamic damping only, eq. (6); and structural damping only, eq. (7)) will be used subsequently in a discussion of the validity of some of the assumptions required in their development.

RESULTS AND DISCUSSION

The data of the present investigation are presented in summary form in figure 6 where the root-mean-square bending moments measured for each of the models are plotted as functions of velocity for angles of attack from 0° to 20°. Examination of figure 6 permits a few general observations.

The root-mean-square bending moments increase rapidly with velocity in a somewhat erratic manner; that is, they do not vary in a simple exponential relationship as would be indicated by straight lines on the logarithmic plot. This result can be expected when it is realized that several of the important buffeting parameters such as Mach number and reduced frequency vary with the velocity. In order to examine the effects of stiffness on the results it will be necessary to make comparisons between the three models at the same values of velocity and angle of attack.

Accordingly, the measured values of buffeting loads for the magnesium and aluminum models at selected values of velocity and angle of attack are plotted against the calculated values for these same conditions in figure 7. Figure 7 contains three parts wherein the calculated values were obtained by the three relations discussed in the section "Method of Analysis." The calculated values in figure 7(a) were obtained by use of equation (4) including aerodynamic and structural damping whereas figures 7(b) and 7(c) refer to calculated values obtained from the relationships for aerodynamic damping only and structural damping only. Plotting the data in this manner permits an examination of the accuracy with which the relationships can be used to extrapolate from the buffeting loads measured on the stiff steel wing to those to be expected for the more flexible aluminum and magnesium. Although there are other factors involved, this extrapolation from a stiff wing to a flexible wing is roughly comparable to the extrapolation of buffeting loads from wind-tunnel model results to those for an airplane in flight.

Examination of figure 7 reveals several interesting results. The use of the complete expression including both aerodynamic and structural damping results in extrapolated values of buffeting loads which are about 25 percent too large at the higher levels of buffeting. The use of the relation for aerodynamic damping only, which, of course, is not as exact within the framework of assumptions as the more complete expression, yields extrapolated values of the loads which agree very well with the measured loads. The values calculated by use of the expression for structural damping are high by a factor of about 2.

The discrepancy between measured and extrapolated buffeting loads for the case where both aerodynamic and structural damping are considered could be due to either or both of the possible phenomena mentioned previously - namely, a nonlinearity of the aerodynamic damping or an alteration of the exciting force caused by the motion of the wing during buffeting.

Examination of the data suggests a speculative explanation of this behavior. The point of view is taken that buffeting is due to the forces generated by randomly shed vortices which, in turn, are associated with the shearing action of the separated boundary layer. If this concept is applied to a very flexible lifting surface, it would seem that the motion of the surface might tend to decrease the strength of the vorticity producing the motion. In other words, the tendency of the flexible wing to move with the driving force might tend to reduce those forces relative to the forces acting on a rigid or nonmoving surface. These arguments are supported to some extent by the data shown in figure 7 when it is observed that the linear theory seems to apply better at low buffeting levels and seems to predict somewhat more accurately the buffeting loads for the more rigid aluminum wing than it does for the magnesium wing.

The question of the importance of this relieving effect in the extrapolation of buffeting loads measured on relatively stiff wind-tunnel models to those to be expected on an airplane in flight must consider the current "state of the art" of buffeting predictions. The indications of the present investigation are that extrapolation based on the calculated relationships including both aerodynamic and structural damping might be conservative by about 25 percent. Such a prediction probably still would be superior to predictions available only a few years ago. On the other hand, a prediction based on aerodynamic damping only, which apparently contains compensating inaccuracies, would provide an even closer approximation at least for the conditions considered in this investigation. Furthermore, such extrapolation would require knowledge only of certain geometric and elastic properties and operating conditions. The actual values of the coefficient of aerodynamic damping which, in general, are not available and are difficult to obtain are not required.

CONCLUDING REMARKS

An examination of the validity of some of the assumptions used in the analysis of buffeting loads has been made by means of wind-tunnel tests of models differing in stiffness. A linear analysis predicts buffeting loads which are high by about 25 percent. This difference may be associated with a relieving effect which causes flexible wings to generate smaller buffeting forces than a more rigid wing. A prediction based on aerodynamic damping only, which apparently contained compensating inaccuracies, provided values of buffeting loads which were closer to the measured values than those predicted by the more complete analysis including both structural and aerodynamic damping.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., March 17, 1959.

APPENDIX A

DERIVATION OF EQUATIONS GOVERNING

BUFFET RESPONSE OF A WING

By Don D. Davis, Jr.

In deriving the buffet equations, the procedure will be to determine the normal modes of the wing, to set up the equation for a steady-state forced vibration by Lagrange's method, to solve this equation in order to determine the admittance of the vibrating system, and then to apply the methods of generalized harmonic analysis to determine the response of the system to a random (buffet) input.

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The normal modes of vibration can be determined from the structural characteristics of the wing as described by certain matrices. (See refs. 9, 10, or 11.) For analysis, the wing is divided into a suitable group of elements, each of which is associated with a particular point in the plane of the wing. The elastic properties of the wing are contained in a square matrix of flexibility-influence coefficients, which can be determined by analysis of the structure or by direct measurement. If $\{P\}$ is a set of static loads and $\{z\}$ is a corresponding set of displacements, then

$$\{z\} = [A]\{P\}$$

where $[A]$ is the matrix of flexibility-influence coefficients. The inertia properties of the wing are described by a diagonal matrix, each element of which is the mass associated with an element of the wing. This matrix is denoted by $\lceil M \rfloor$. The matrix $[U] = [A]\lceil M \rfloor$ is called the dynamic matrix.

The matrix equation

$$\{z\} = \omega^2 [U] \{z\}$$

is solved to obtain the frequencies and shapes of the normal modes of vibration (ref. 10, p. 169). The frequency of the nth mode will be written ω_n and the column matrix containing the associated normalized set of deflections will be written $\left\{\varphi^{(n)}\right\}$.

The displacement of the mth element of the vibrating wing can be written in terms of a series utilizing the normal modes:

$$z_m = \sum_n r_n \varphi_m^{(n)}$$

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where the terms r_n are functions of time. The kinetic energy of the vibrating system is then (ref. 10)

$$T = \frac{1}{2} \sum_n M_n \dot{r}_n^{\,2}$$

where

$$M_n = \sum_m m_m \left(\varphi_m^{(n)}\right)^2$$

and the terms m_m are the elements of the inertia matrix $\left[M\right]$. The elastic strain energy E is (refs. 10 and 11)

$$E = \frac{1}{2} \sum_n \omega_n^{\,2} M_n r_n^{\,2}$$

These expressions for the kinetic and potential energies, when inserted in Lagrange's equation, yield the equation of motion for the nth modes:

$$M_n \ddot{r}_n + \omega_n^{\,2} M_n r_n = \sum_m P_m \varphi_m^{(n)} \qquad \text{(A1)}$$

where P_m represents the forces, other than inertial and elastic, that act on the element m.

The results of several investigations (refs. 5 and 12) have shown that, in many instances of wing buffet, most of the energy in the power spectrum of buffet bending moment is concentrated at frequencies in the vicinity of the natural frequency of the first mode. Normally the first mode is well separated from the higher modes and, as a result, the response of the higher modes at the first mode frequency is very small. Attention can be confined, therefore, to a study of the first mode. Equations (A1) then reduce to the single equation

$$M_1\ddot{r}_1 + \omega_1{}^2M_1r_1 = \sum_m P_m\varphi_m^{(1)} \tag{A2}$$

One of the forces that contributes to P_m is the pressure fluctuation that causes the buffet; this pressure fluctuation is called the exciting force. The force on element m is $\Delta p_m s_m$ and the corresponding generalized force on the wing is

$$N_1 = \sum_m \Delta p_m s_m \varphi_m^{(1)}$$

It is convenient to define what might be termed a generalized normal-force coefficient for the first mode:

$$C_{N,1} = \frac{N_1}{qS_1} \tag{A3}$$

where

$$S_1 = \sum_m s_m\varphi_m^{(1)} \tag{A4}$$

Another force that contributes to P_m is the aerodynamic force due to the motion of the wing. For simple harmonic motion, this force for an element m of the wing is of the form

$$a_m\ddot{r}_1 + b_m\dot{r}_1 + c_mr_1$$

with the corresponding generalized force being

$$\ddot{r}_1 \sum_m a_m\varphi_m^{(1)} + \dot{r}_1 \sum_m b_m\varphi_m^{(1)} + r_1 \sum_m c_m\varphi_m^{(1)}$$

In this simplified treatment of the buffet phenomena, the first and last terms of this generalized force are assumed to be negligible in comparison with $M_1\ddot{r}_1$ and $\omega_1{}^2M_1r_1$, respectively. Further consideration is given to the second term, which arises from the aerodynamic forces that oppose the vertical velocity of each element m. The resulting pressure difference has the form

$$\Delta p_m = -qk_m \frac{\dot{z}_m}{V} = -q\,\frac{k_m}{V}\,\dot{r}_1\varphi_m^{(1)}$$

where $\dfrac{\dot{z}_m}{V}$ is an effective angle of attack and k_m is a constant of the nature of a local lift-curve slope that depends on the plan form and mode shape. The minus sign signifies that the pressure opposes the motion. The corresponding generalized force is

$$-L_1\dot{r}_1 = -q\,\frac{\dot{r}_1}{V}\sum_m k_m s_m\left(\varphi_m^{(1)}\right)^2$$

It is convenient to define what might be called a generalized lift-curve slope for the first mode:

$$C_{L_{\alpha,1}} = \frac{\displaystyle\sum_m k_m s_m\left(\varphi_m^{(1)}\right)^2}{S_2} \tag{A5}$$

where

$$S_2 = \sum_m s_m\left(\varphi_m^{(1)}\right)^2 \tag{A6}$$

so that

$$L_1\dot{r}_1 = C_{L_{\alpha,1}}\,\frac{\dot{r}_1}{V}\,qS_2 \tag{A7}$$

The equation of motion can now be written as

$$M_1\ddot{r}_1 + L_1\dot{r}_1 + \omega_1{}^2M_1r_1 = N_1$$

The term $L_1\dot{r}_1$ is the generalized aerodynamic damping force. Structural damping can be included by adding a term $ig_1\omega_1^2M_1r_1$ (ref. 10, p. 197). For a sinusoidal force $N_1 = N \sin \omega t$ the equation of motion is then

$$M_1\ddot{r}_1 + L_1\dot{r}_1 + \left(1 + ig_1\right)\omega_1^2M_1r_1 = N \sin \omega t$$

The steady-state solution of this equation is

$$r_1 = \frac{N}{M_1\omega_1^2} \frac{\sin(\omega t - \beta)}{\left[\left(1 - \frac{\omega^2}{\omega_1^2}\right)^2 + \left(\frac{2\gamma_1\omega}{\omega_1} + g_1\right)^2\right]^{1/2}}$$

where $\gamma_1 = \frac{L_1}{2M_1\omega_1}$ and β is the phase angle by which the displacement lags the force. For use in generalized harmonic analysis of buffeting, the square of the absolute value of the admittance is required:

$$\left|A(\omega)\right|^2 = \frac{1}{M_1^2\omega_1^4\left[\left(1 - \frac{\omega^2}{\omega_1^2}\right)^2 + \left(\frac{2\gamma_1\omega}{\omega_1} + g_1\right)^2\right]} \tag{A8}$$

According to the principles of generalized harmonic analysis, the response of this system to a random input $\Phi_N(\omega)$ is

$$\Phi_{r_1}(\omega) = \Phi_N(\omega)\left|A(\omega)\right|^2$$

The mean-square value is given by

$$\overline{r_1^2} = \int_0^\infty \Phi_N(\omega)\left|A(\omega)\right|^2 d\omega$$

In the case of a lightly damped system, the response is concentrated in a narrow frequency band near ω_1. In that band the response is approximately

$$\Phi_{r_1}(\omega) \approx \Phi_N(\omega_1)\left|A(\omega)\right|^2$$

if the input spectrum is reasonably smooth. Flight-test results (ref. 5) show that all but a very small part of the response power for a buffeting wing is found in the frequency band near ω_1 and, therefore, the mean-square response will be approximately

$$\overline{r_1^2} \approx \Phi_N(\omega_1) \int_0^\infty \left|A(\omega)\right|^2 d\omega$$

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This equation cannot be readily integrated in closed form; however, for small values of structural and aerodynamic damping a satisfactory approximation of the integral gives

$$\overline{r_1^2} \approx \Phi_N(\omega_1) \frac{\pi\omega_1}{M_1^2\omega_1^4 4\left(\gamma_1 + \frac{g_1}{2}\right)} \tag{A9}$$

Assume now that a strain gage has been mounted on the wing at any point that experiences strain fluctuations during first-mode vibration of the wing. When the wing vibrates in the first mode, the elongation sensed by the gage, and hence the gage output, will be directly proportional to the amplitude r_1 of the vibration. Hence, r_1 can be determined with a properly calibrated strain gage. (The case where the wing is vibrating in several modes is not considered herein. Such a case involves solutions of the set of equations (A1) rather than of a single equation of the set.) Thus, the power spectrum $\Phi_{r_1}(\omega)$ and the mean-square value $\overline{r_1^2}$ of the vibration amplitude can be obtained from analysis of the strain-gage output.

In the case where a wing can be treated as a simple beam, the strain gages can be calibrated in terms of the beam bending moment, and a relationship can be derived between the bending moment and the generalized input force for the first-mode bending of the wing. (See ref. 5.) The resulting equation for the mean-square bending moment σ^2 is

$$\sigma^2 = y_{m_1}^2 \Phi_N(\omega_1) \frac{\pi\omega_1}{4\left(\gamma_1 + \frac{g_1}{2}\right)} \tag{A10}$$

where

$$y_{m_1}^2 = \frac{M_{m,1}^2}{M_1^2}$$

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L-200

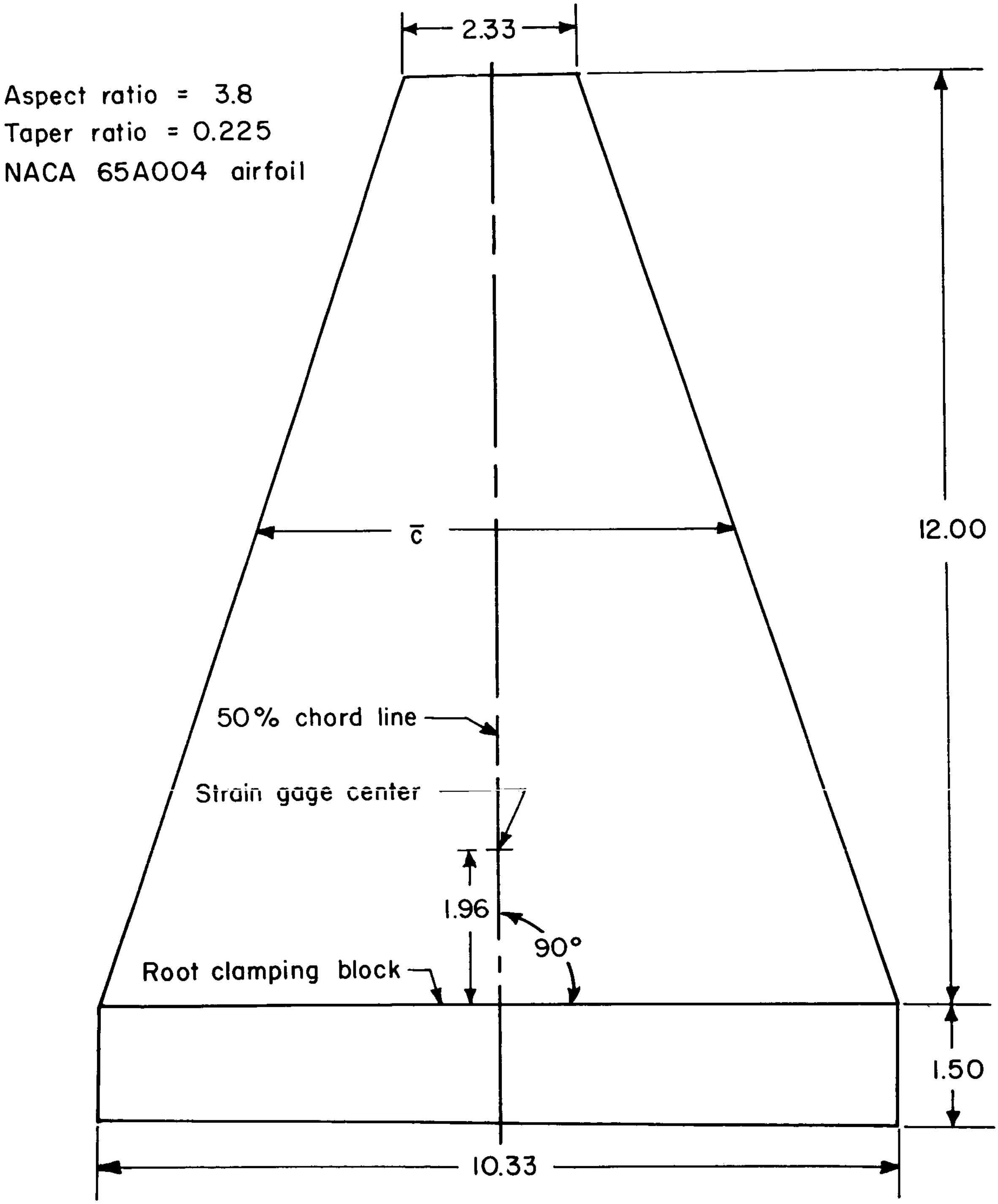

Figure 1.- Sketch of model.

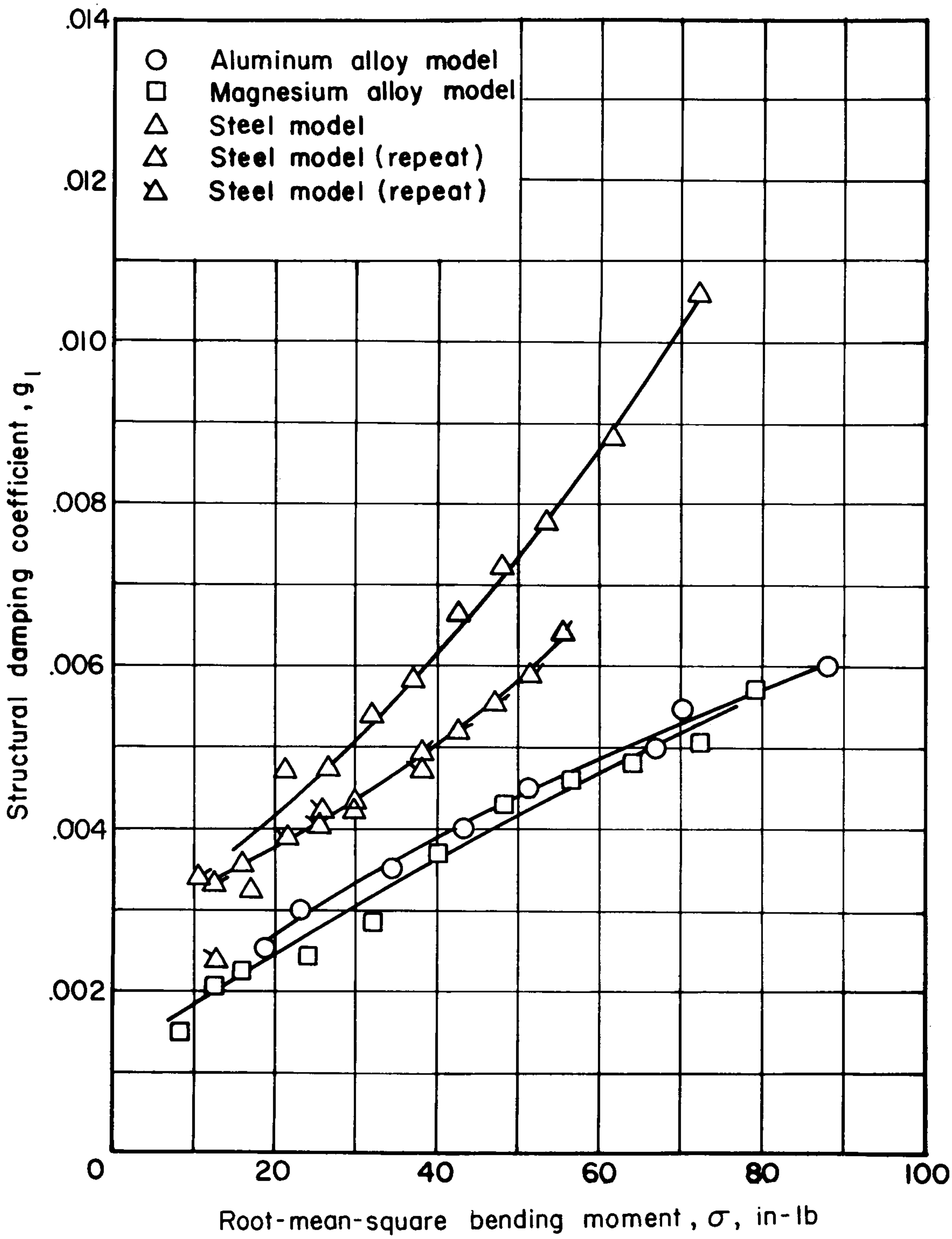

Figure 2.- Variation of structural damping in still air with root-mean-square bending moment.

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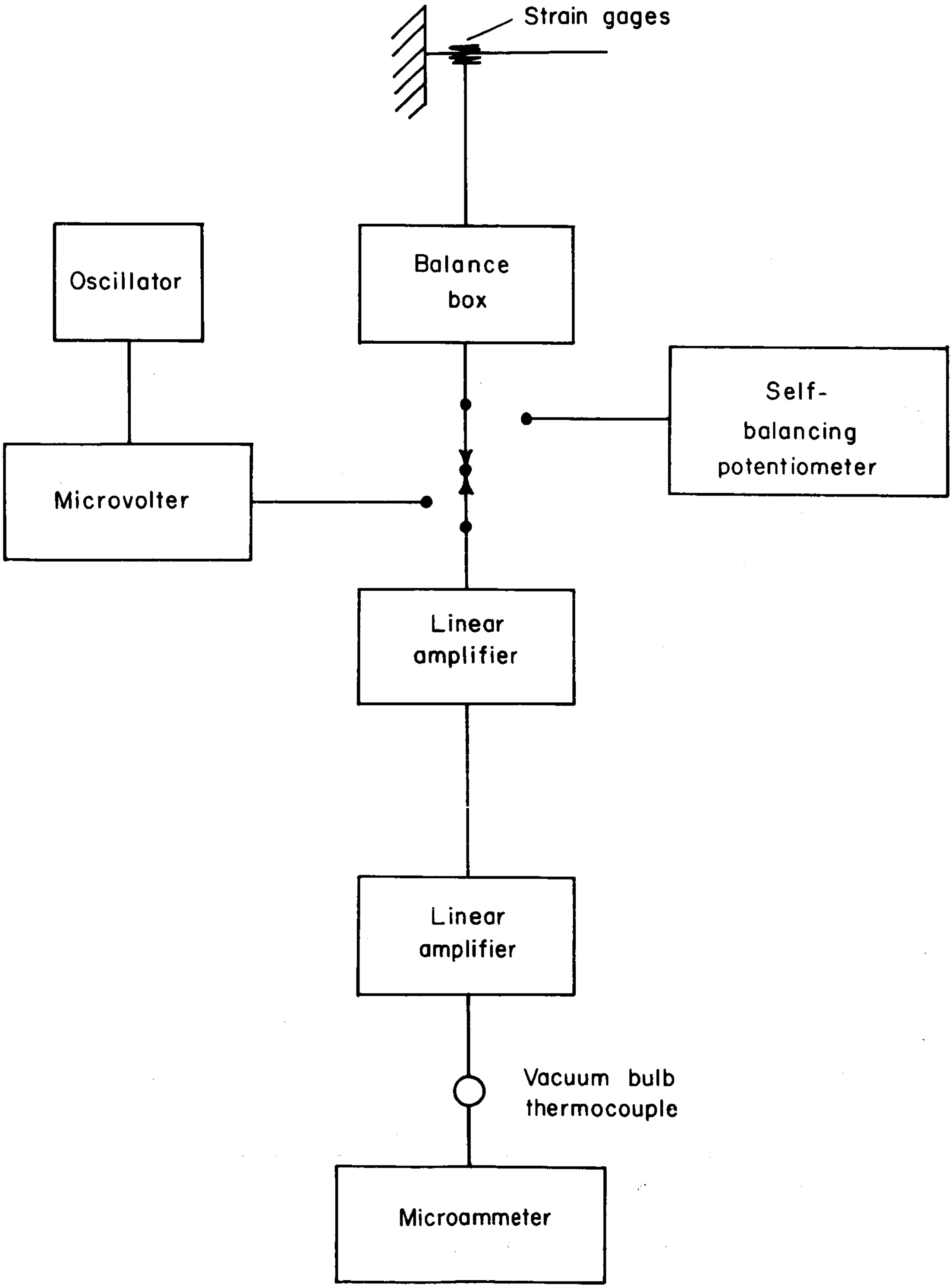

Figure 3.- Schematic diagram of instrumentation.

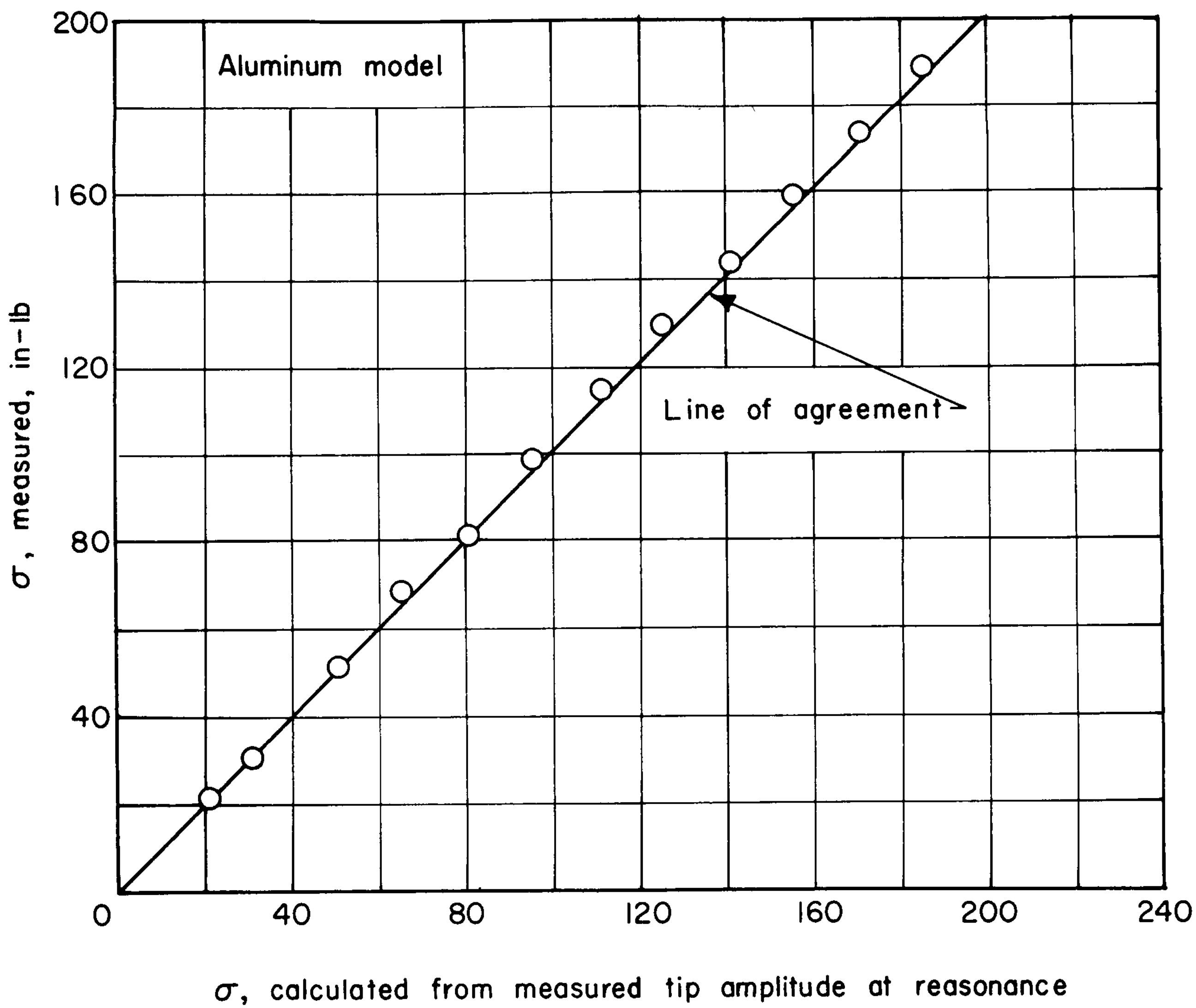

Figure 4.- Verification of calibration procedure.

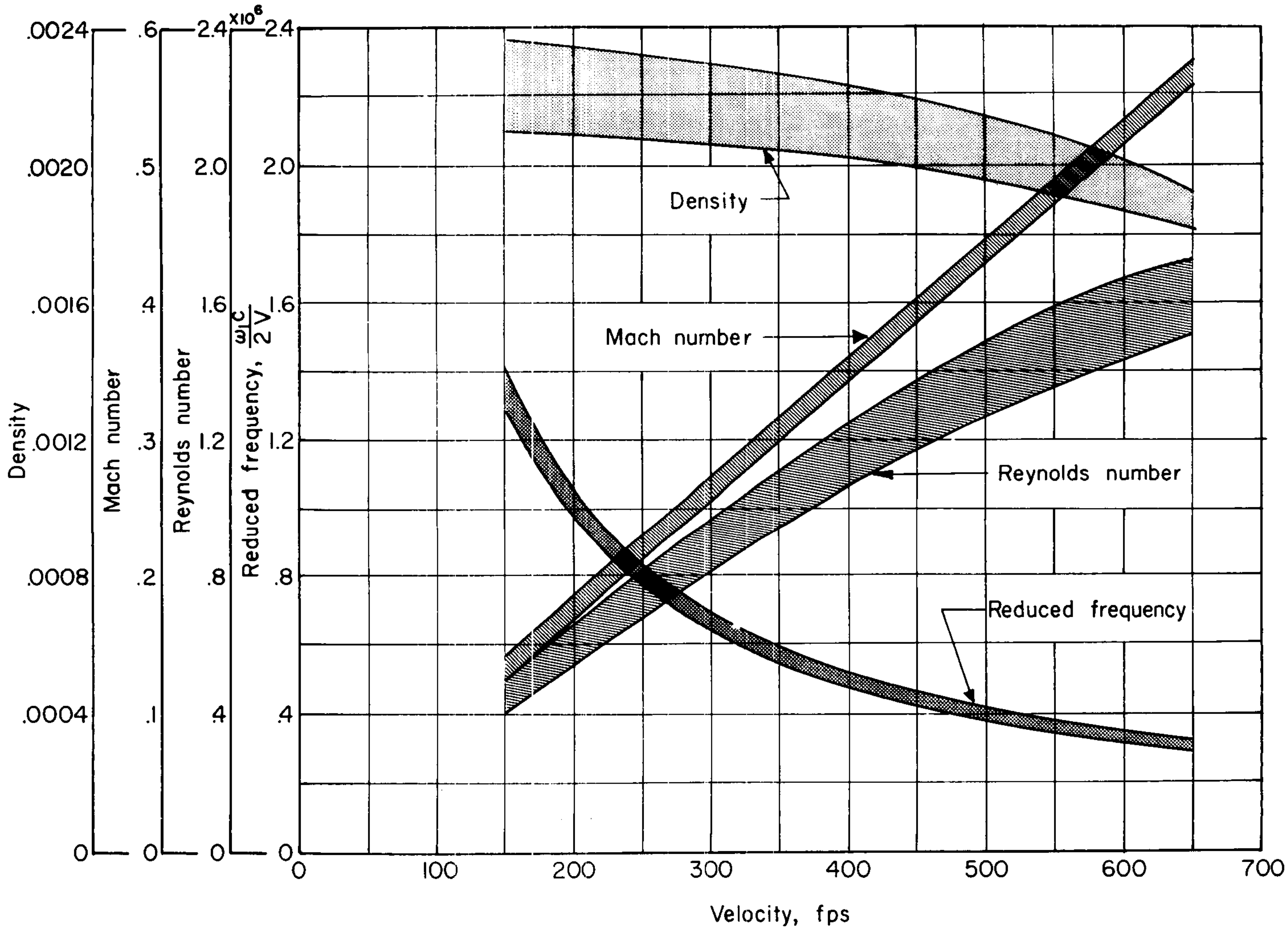

Figure 5.- Variation of density, Mach number, Reynolds number, and reduced frequency with velocity.

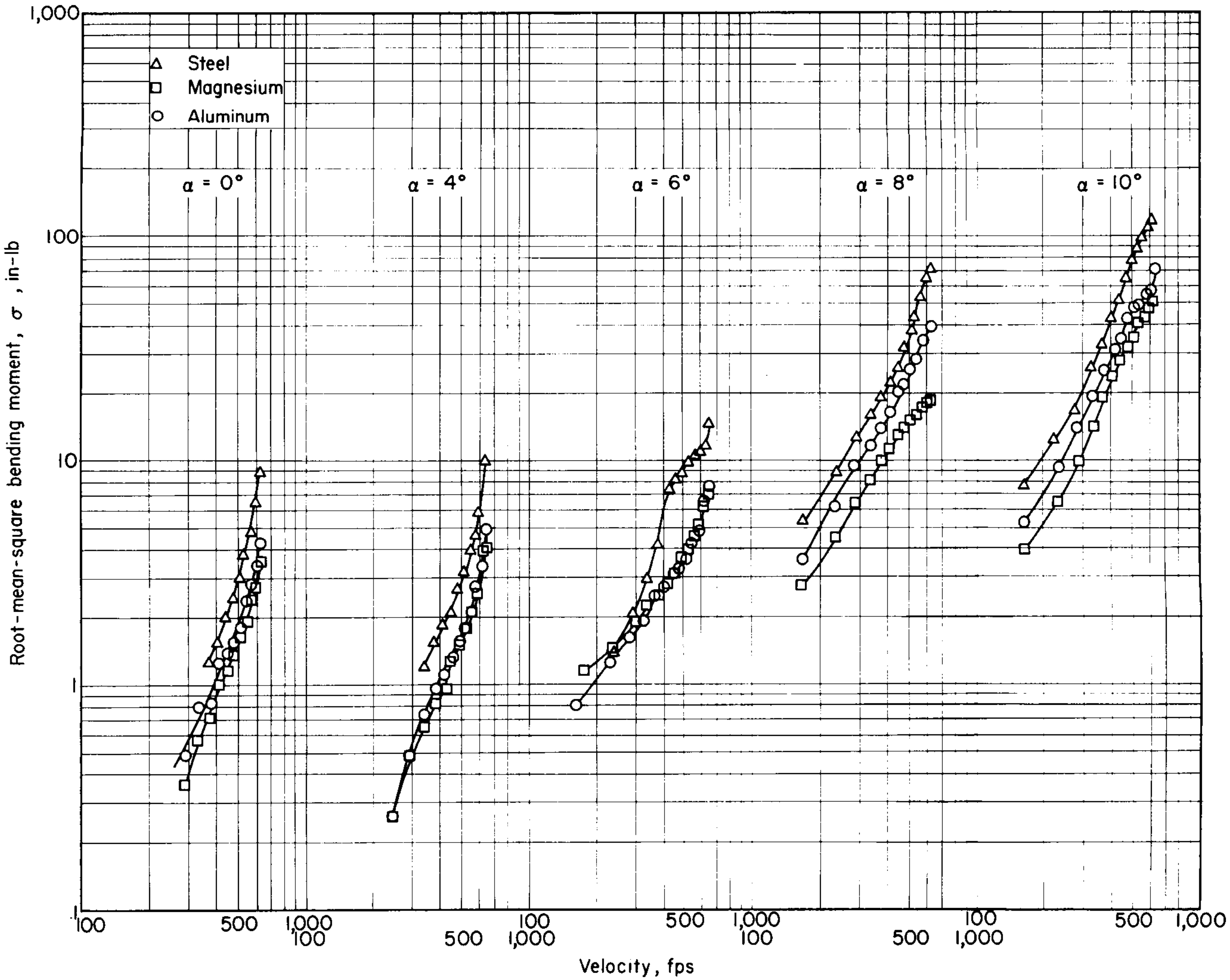

Figure 6.- Comparison of root-mean-square bending moment for three models at various angles of attack.

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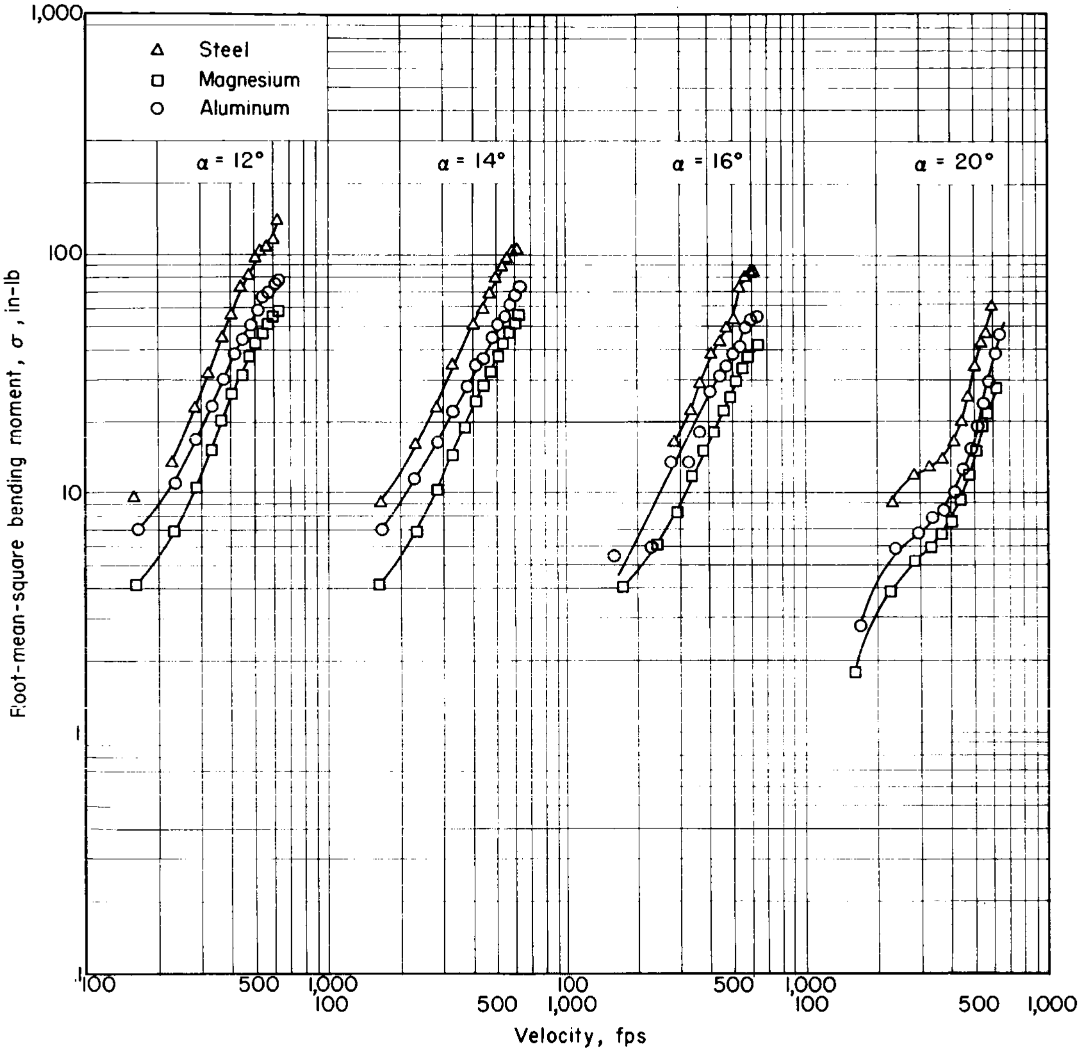
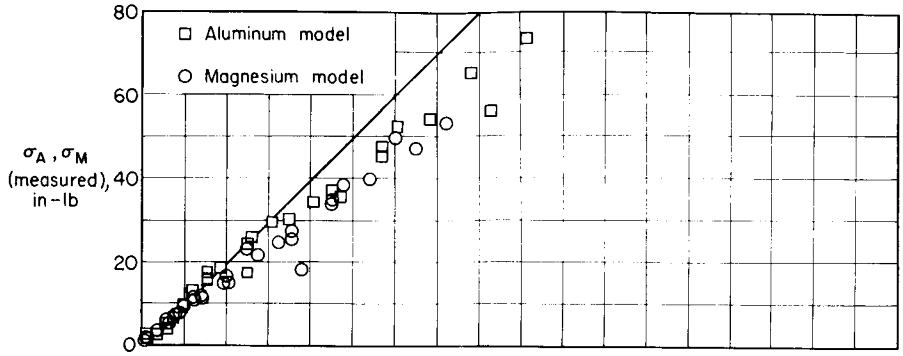
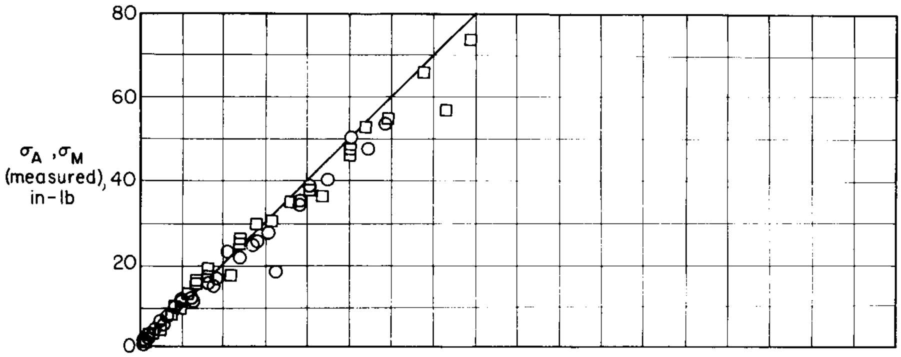


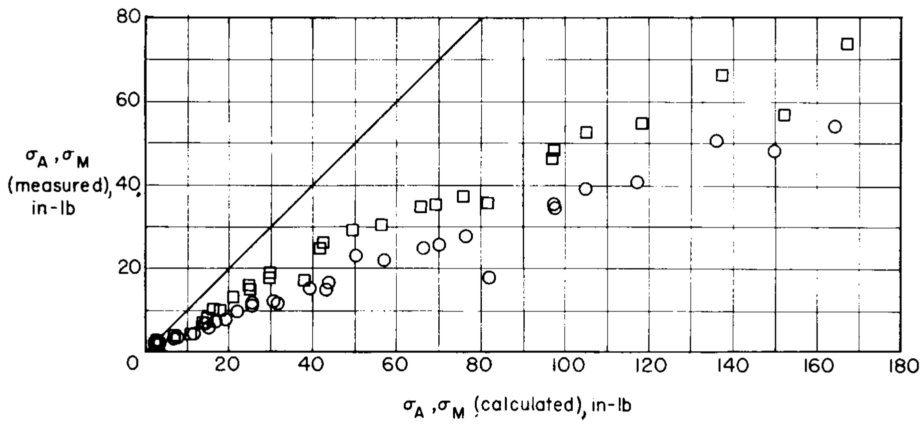
Figure 6.- Concluded.



(a) Aerodynamic and structural damping.



(b) Aerodynamic damping only.



(c) Structural damping only.

Figure 7.- Comparison of measured and calculated root-mean-square bending moments.